Will it Happen?

This problem gives you the chance to:
• describe events as likely or unlikely as appropriate
• find the numerical probability of various outcomes of rolling a number cube

What does the future hold?
Select just one of these five words and write it after the following statements.

impossible	unlikely	equally likely	likely	certain

1. a. If today is Monday, tomorrow will be Tuesday. ___________________

   b. Today you will meet President Lincoln on the way home from school. ___________________

   c. When you flip a coin it will land head up. ___________________

2 a. When you roll a number cube with faces numbered 1, 2, 3, 4, 5, 6, what is the **numerical** probability of getting the number 4?

____

   b. When you roll a number cube with faces numbered 1, 2, 3, 4, 5, 6, what is the **numerical** probability it will land on an odd number?
   Explain how you figured it out. ___________________

____

____

3. The faces of one red number cube and one blue number cube are labeled 1, 3, 5, 7, 9, 11.
   The two cubes are rolled and the results are added.

   What is the **numerical** probability of getting a total of 20? _____________
   Show how you figured it out.

8

Will it Happen?		Rubric
• • The core elements of performance required by this task are: • describe events as likely or unlikely as appropriate • find the numerical probability of various outcomes of rolling a number cube. • Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Gives correct answers: a. **certain** b. **impossible** c. **equally likely** *Partial credit* 2 correct	2 (1)	2
2.a. Gives correct answer **1/6** b. Gives correct answer: **3/6 or 1/2** Gives correct explanation such as: there are 3 of 6 equally likely possibilities	1 1 1	3
3. Gives correct answer **2/36** or **1/18** Shows work such as: there are 36 equally likely outcomes. and 20 = 9 + 11 and 11 + 9 *Partial credit* Allow partial credit for some correct work.	1 2 (1)	3
Total Points		**8**

Will It Happen?

Work the task. Look at the rubric. What are some of the big ideas about probability that are being assessed in this task? What do you think might be problematic for students?

In part one, how many of your students chose unlikely for part b?_________________
What might they have been thinking?
Now look at student work for part 2a. How many of your students put:

1/6	2/3	Equally likely	Likely	Unlikely	Other

Why might students choose 2/3? Why might students use words instead of numbers? What is the logic behind choosing equally likely or unlikely? What are the different misconceptions for each answer?

Now look at part 2b of the task. How many of your students put:

3/6 or 1/2	1/3 or 2/6	Likely	Equally likely	3	Other

What are the misconceptions shown by these responses?

2/36	4/12	1/12	2/12	1/20	Unlikely	Likely	1/36	11 & 9

How were students thinking about sample space? Were they thinking about 1 die, 2 dice, the total from adding the two dice or the total possible outcomes?

What experiences have your students had with probability?
How might probability be reinforced in the curriculum when teaching other concepts?

What kind of experiences do students bring to the classroom from previous years? How should their knowledge of probability deepen at this grade level?

What is your role in helping to prepare students for the high school exit exam?

Looking at Student Work on Will it Happen?

Student A is able to describe probabilities in words and numbers. Student A is able to use an organized list to define the sample space in part 3. Student A is also able to verify the total number of outcomes for 2 dice using multiplication.

Student A

impossible	unlikely	equally likely	likely	certain

1. a. If today is Monday, tomorrow will be Tuesday. certain

   b. Today you will meet President Lincoln on the way home from school. impossible

   c. When you flip a coin it will land head up. equally likely

2. a. When you roll a number cube with faces numbered 1, 2, 3, 4, 5, 6, what is the **numerical** probability of getting the number 4?

   1 out of 6

   b. When you roll a number cube with faces numbered 1, 2, 3, 4, 5, 6, what is the **numerical** probability it will land on an odd number? 1 2 1 2 1 2
   Explain how you figured it out. 3 out of 6

   I figured this out by counting the odd numbers (3) and counting the total (6) and got 3 out of 6.

3. The faces of one red number cube and one blue number cube are labeled 1, 3, 5, 7. 9. 11.
   The two cubes are rolled and the results are added.

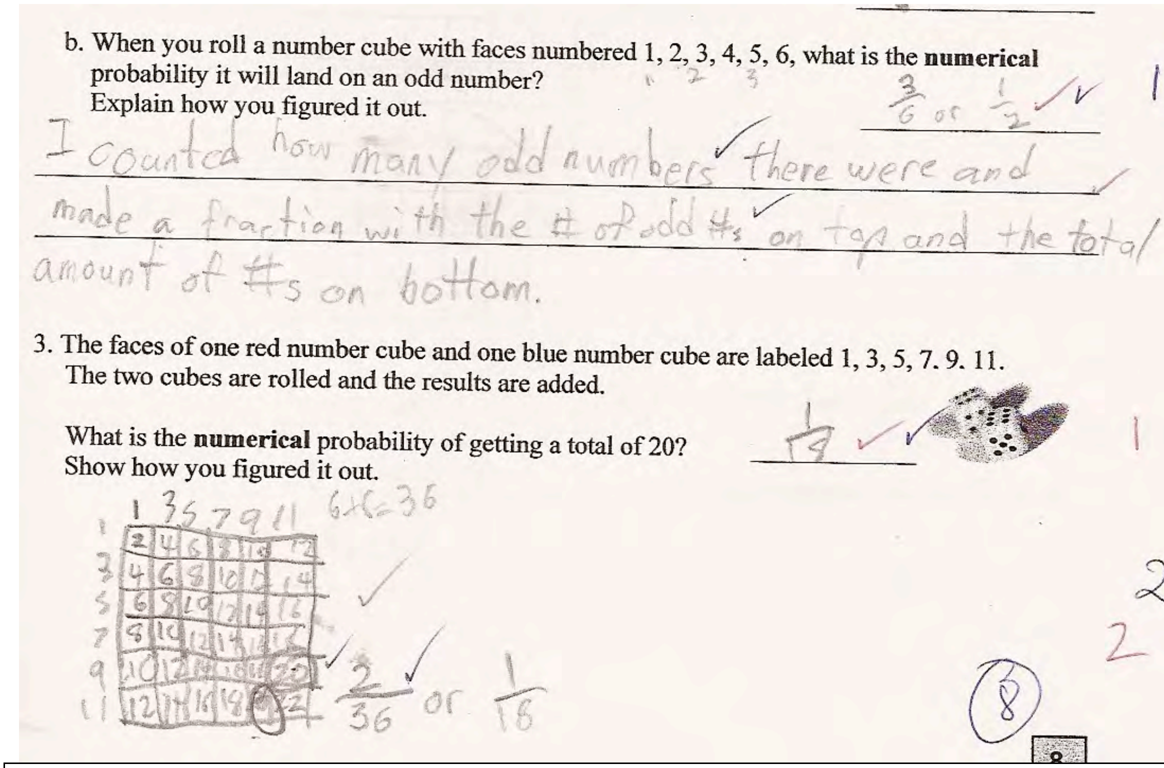
   What is the **numerical** probability of getting a total of 20? 2 out of 36
   Show how you figured it out.

1+3	3+1	5+5	7+1	9+1	11+1
1+5	3+3	5+1	7+3	9+3	11+3
1+7	3+5	5+3	7+5	9+5	11+5
1+1	3+7	5+7	7+7	9+7	11+7
1+9	3+9	5+9	7+9	9+9	11+9
1+11	3+11	5+11	7+11	9+11	11+11

$6 \times 6 = 36$

Student B uses a diagram to find the sample space in part 3 and confirms the total outcomes with multiplication. Notice the careful use of language in describing the numerical probability in part 2b. Watch how the use of language changes for students with lower total scores. *How does understanding a working definition effect the type of thinking in part 3?*

Student B



b. When you roll a number cube with faces numbered 1, 2, 3, 4, 5, 6, what is the **numerical** probability it will land on an odd number?
Explain how you figured it out.

$\frac{3}{6}$ or $\frac{1}{2}$

I counted how many odd numbers there were and made a fraction with the # of odd #s on top and the total amount of #s on bottom.

3. The faces of one red number cube and one blue number cube are labeled 1, 3, 5, 7. 9. 11.
The two cubes are rolled and the results are added.

What is the **numerical** probability of getting a total of 20?
Show how you figured it out.

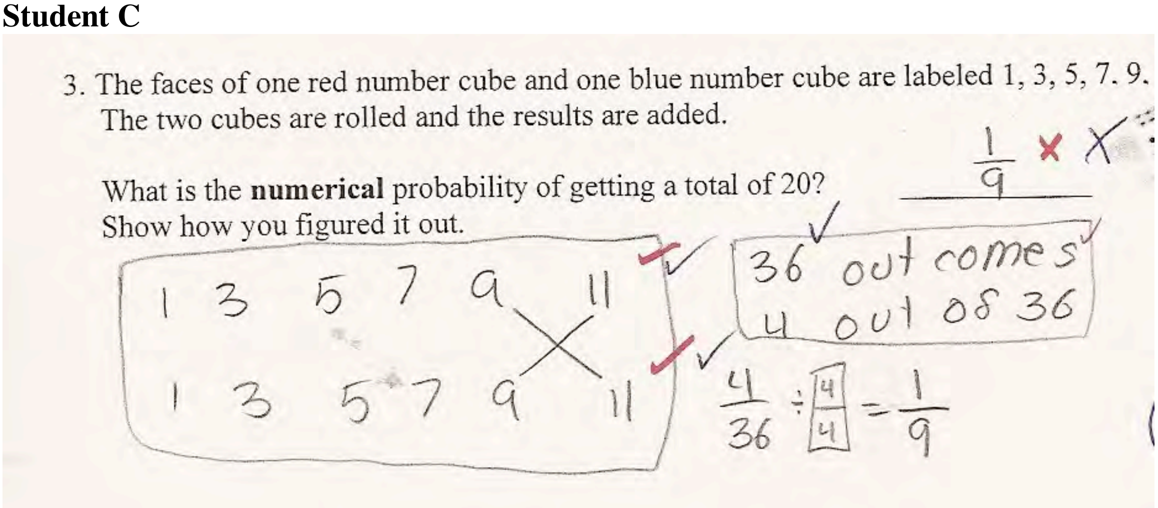
$\frac{1}{18}$

6×6=36

$\frac{2}{36}$ or $\frac{1}{18}$

Many students were successful in solving parts 1 and 2, but struggled with the thinking in part 3. Student C is able to figure out the number of total outcomes and see that there are 2 combinations that will add to 20. However, the student confuses the numbers on the dice with possible outcomes, similar to counting the odd numbers in part 2b. He should have been counting the number of combinations.

Student C



3. The faces of one red number cube and one blue number cube are labeled 1, 3, 5, 7. 9.
The two cubes are rolled and the results are added.

What is the **numerical** probability of getting a total of 20?
Show how you figured it out.

$\frac{1}{9}$

1 3 5 7 9 11

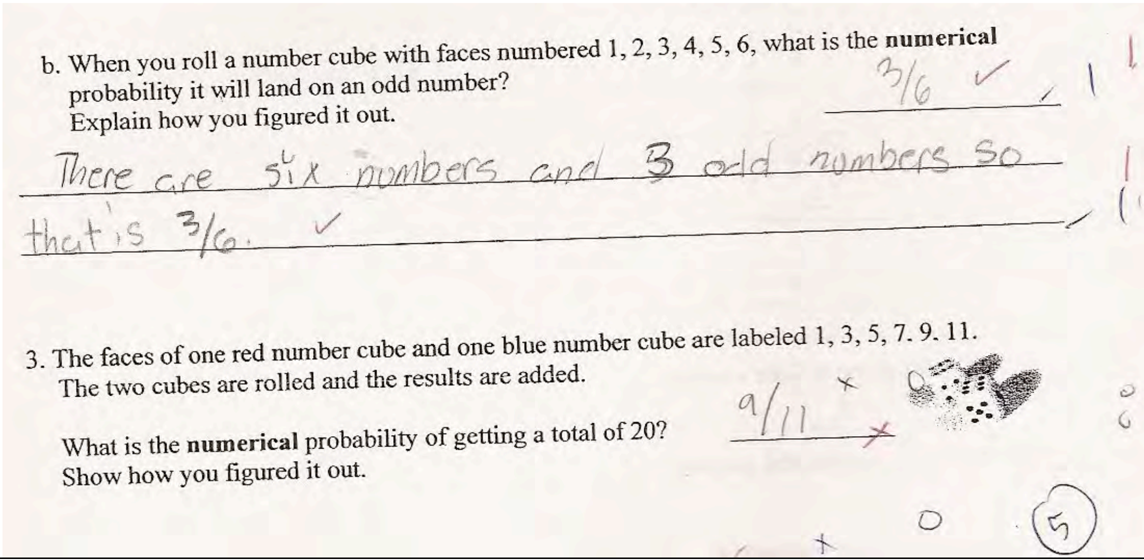
1 3 5 7 9 11

36 out comes
4 out of 36

$\frac{4}{36} \div \boxed{\frac{4}{4}} = \frac{1}{9}$

Student D also receives credit for parts 1 and 2. The student uses the numbers on the dice, which sum to 20 to make the probability. Look at the explanation in 2b. *How might the imprecision here have contributed to making errors in 3?*

Student D



b. When you roll a number cube with faces numbered 1, 2, 3, 4, 5, 6, what is the **numerical** probability it will land on an odd number?
Explain how you figured it out.

3/6

There are six numbers and 3 odd numbers so that is 3/6.

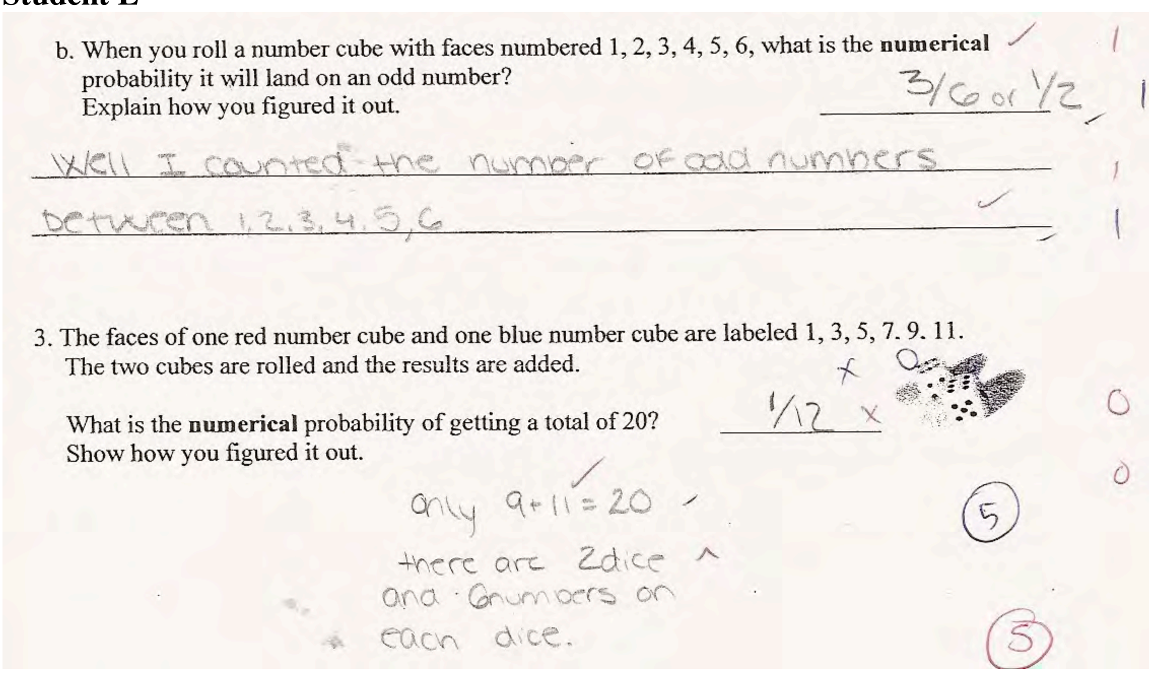
3. The faces of one red number cube and one blue number cube are labeled 1, 3, 5, 7. 9. 11.
The two cubes are rolled and the results are added.

What is the **numerical** probability of getting a total of 20?
Show how you figured it out.

9/11

Student E again uses a weak explanation in part 2b. The student finds a combination to make 20, but then thinks of the sample space as the total numbers of the 2 dice rather than the total number of outcomes. *How does the concept of sample space change when going from one object or set to combining options? How do we help students grasp this significant change? What questions might we ask to make these changes more explicit?*

Student E



b. When you roll a number cube with faces numbered 1, 2, 3, 4, 5, 6, what is the **numerical** probability it will land on an odd number?
Explain how you figured it out.

3/6 or 1/2

Well I counted the number of odd numbers between 1, 2, 3, 4, 5, 6

3. The faces of one red number cube and one blue number cube are labeled 1, 3, 5, 7. 9. 11.
The two cubes are rolled and the results are added.

What is the **numerical** probability of getting a total of 20?
Show how you figured it out.

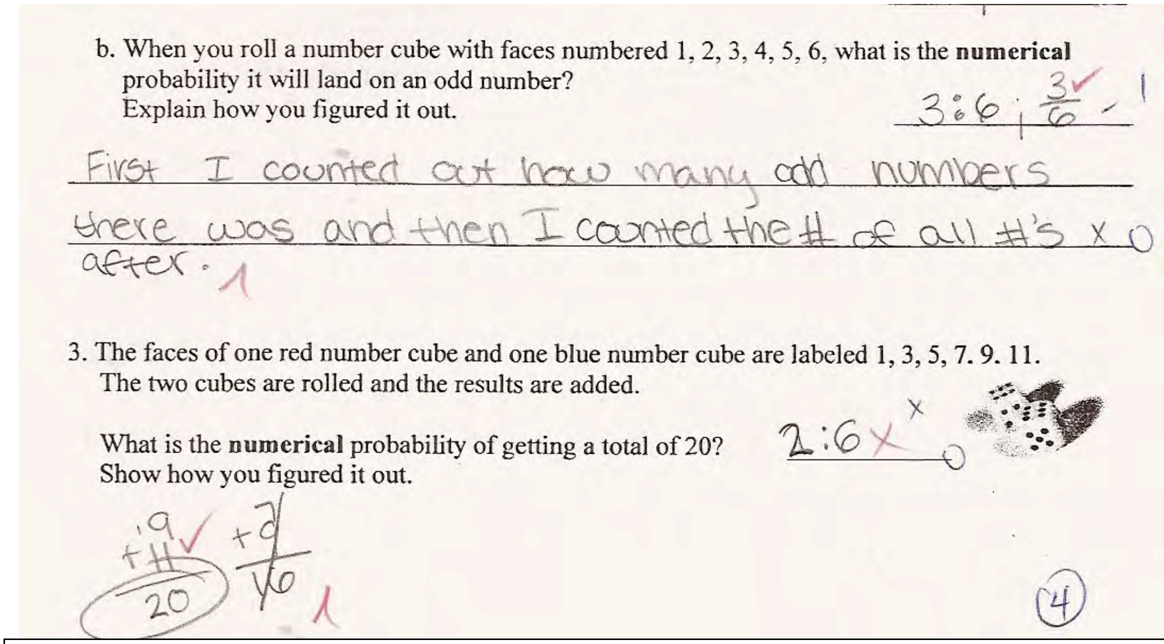
1/12

only 9+11=20

there are 2dice and 6numbers on each dice.

Student F has a good concept of finding numerical probabilities for 1 die. However the student does not think about combining the rolls of two dice in part 3, but treats the situation as an event with one die, using the logic of finding the odd numbers on one die.

Student F



b. When you roll a number cube with faces numbered 1, 2, 3, 4, 5, 6, what is the **numerical** probability it will land on an odd number?
Explain how you figured it out. 3:6 | 3/6 1

First I counted out how many odd numbers there was and then I counted the # of all #'s after. x 0 1

3. The faces of one red number cube and one blue number cube are labeled 1, 3, 5, 7. 9. 11.
The two cubes are rolled and the results are added.

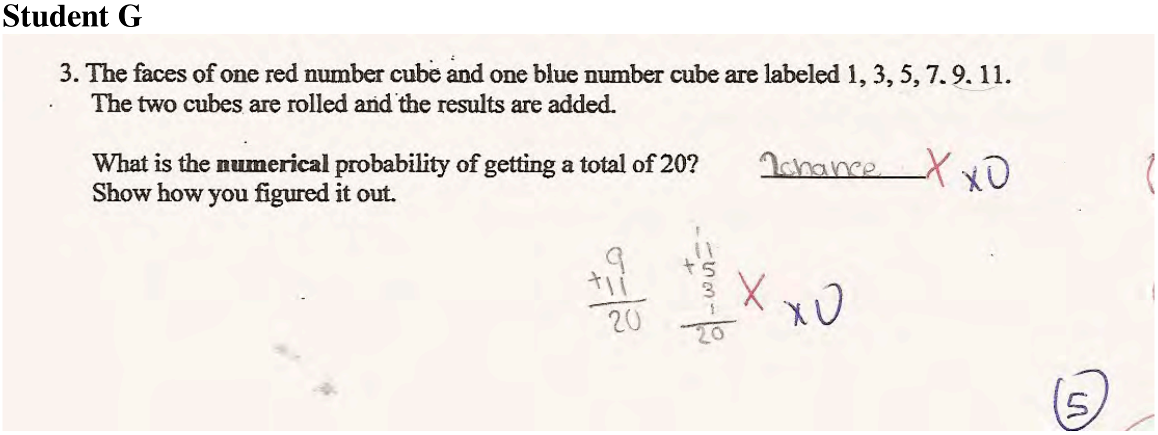
What is the **numerical** probability of getting a total of 20? 2:6 x 0
Show how you figured it out.

9 + 11 = 20 1

4

Student G is only looking at the possibilities and loses the idea of context. The student finds 4 numbers that will add to 20 and considers that as a possible outcome for rolling 2 dice.

Student G



3. The faces of one red number cube and one blue number cube are labeled 1, 3, 5, 7. 9. 11.
The two cubes are rolled and the results are added.

What is the **numerical** probability of getting a total of 20? 1chance x 0
Show how you figured it out.

9 + 11 = 20   11 + 5 + 3 + 1 = 20 x 0

5

Students with lower scores do not know how to find numerical probabilities. Student H says that rolling a 4 is likely. Here the student is not thinking in numerical terms, getting a 4 isn't very likely because there are a lot of other choices; but she is thinking that it is a possible outcome so it might happen. *Do we think about the number sense students need to choose the appropriate words versus the everyday use of the words? How do we help students attach quantitative thinking to the probability language?* Notice that the student does not understand that the numbers on each die do not have to match in part 3.

Student H

impossible	unlikely	equally likely	likely	certain

1. a. If today is Monday, tomorrow will be Tuesday. certain ✓ 1

b. Today you will meet President Lincoln on the way home from school. impossible ✓

c. When you flip a coin it will land head up. equally likely ✓ 2 1

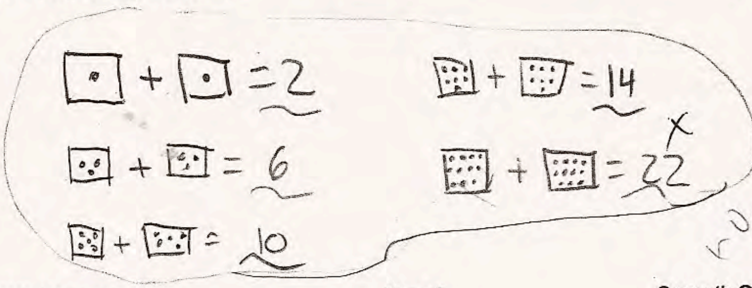
2 a. When you roll a number cube with faces numbered 1, 2, 3, 4, 5, 6, what is the **numerical** probability of getting the number 4? likely X 0 0

b. When you roll a number cube with faces numbered 1, 2, 3, 4, 5, 6, what is the **numerical** probability it will land on an odd number? equally likely X 0
Explain how you figured it out.

I got equally likely because there are 3 odd #s and 3 even #s so it is equally likely because they are equal. ✓ 1 1

3. The faces of one red number cube and one blue number cube are labeled 1, 3, 5, 7. 9. 11.
The two cubes are rolled and the results are added.

What is the **numerical** probability of getting a total of 20? unlikely X 0 0
Show how you figured it out.



0 0 3

There is no 20 so you 8 can't get it numericl

 Page 1 Seventh Grade - Will it Happen?

3

Student I is good at reducing fractions. The student is on the borderline of understanding very simple probability. In part 2b the student can explain how to find the numerical probability for rolling an odd number. However in 2a the student confuses the roll of the die with the number of possible outcomes. In part 3 the student considers only 1 die in defining the sample space. The student here knows to think about outcomes instead of numbers on the die. *How are the three parts using the numerator in slightly different ways?*

Student I

2 a. When you roll a number cube with faces numbered 1, 2, 3, 4, 5, 6, what is the **numerical** probability of getting the number 4?

0 $\frac{4}{6} = \frac{2}{3}$ X

b. When you roll a number cube with faces numbered 1, 2, 3, 4, 5, 6, what is the **numerical** probability it will land on an odd number? Explain how you figured it out.

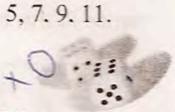
1 $\frac{3}{6} = \frac{1}{2}$ ✓ 2

I know that there are six numbers and only three of them are odd. So, I put the three over six and reduced, to get $\frac{1}{2}$. ✓ 1

3. The faces of one red number cube and one blue number cube are labeled 1, 3, 5, 7, 9, 11. The two cubes are rolled and the results are added.

What is the **numerical** probability of getting a total of 20? Show how you figured it out.

$\frac{1}{6}$ X 0



$11 + 9 = 20$ X 0

1, 3, 5, 7, 9, 11

6

Student J struggles with the quantitative thinking needed to use the probability words correctly. In 1c the student is thinking that heads is likely to come out, but doesn't consider that it is 1 of 2 possible outcomes so it should be equally likely. In part 2 the student does not attempt to use numbers to describe the probabilities. In part 3 the student looks at the numbers needed to make the desired outcomes rather than thinking about probability.

Student J

impossible	unlikely	equally likely	likely	certain

1\. a. If today is Monday, tomorrow will be Tuesday. Certain ✓✓

b. Today you will meet President Lincoln on the way home from school. impossible ✓✓

c. When you flip a coin it will land head up. likely X

2 a. When you roll a number cube with faces numbered 1, 2, 3, 4, 5, 6, what is the **numerical** probability of getting the number 4?

Unlikely X x0

b. When you roll a number cube with faces numbered 1, 2, 3, 4, 5, 6, what is the **numerical** probability it will land on an odd number?
Explain how you figured it out.

equally likely X x0

1, 3, 5, X x0

3\. The faces of one red number cube and one blue number cube are labeled 1, 3, 5, 7, 9, 11.
The two cubes are rolled and the results are added.

What is the **numerical** probability of getting a total of 20? 11 + 9 X
Show how you figured it out. x0

Student Task	Describe events as likely or unlikely. Find the numerical probability of various outcomes of rolling a number cube.
Core Idea 2 Probability	**Apply and deepen understanding of theoretical and empirical probability.** • Represent the sample space for simple and compound events in an organized way. • Represent probabilities as ratios, proportions, decimals or percents. • Determine theoretical and experimental probabilities and use these to make predictions about events.

Mathematics of this task:
- Understand the quantitative thinking needed to use probability words, such as impossible, likely, equally likely, certain, etc.
- Understand the difference between numbers on a die and outcomes
- Describe the numerical probability of a situation
- Understand and calculate the sample space for combined probabilities
- Distinguish between the number of ways an outcome can be obtained numerically and the number of possible outcomes from a situation (9+11= 20 are the numbers that make the desired outcome 20. The 9 could be on either the red or the blue die, so there are two possible ways of making the 9 + 11.)

Based on teacher observations, this is what seventh graders know and are able to do:
- Use words to describe probabilities.
- Write a numerical probability for a single event, like rolling a 4 on a die or rolling an even number.
- Describe how to find the probability for a simple event like rolling an odd number.

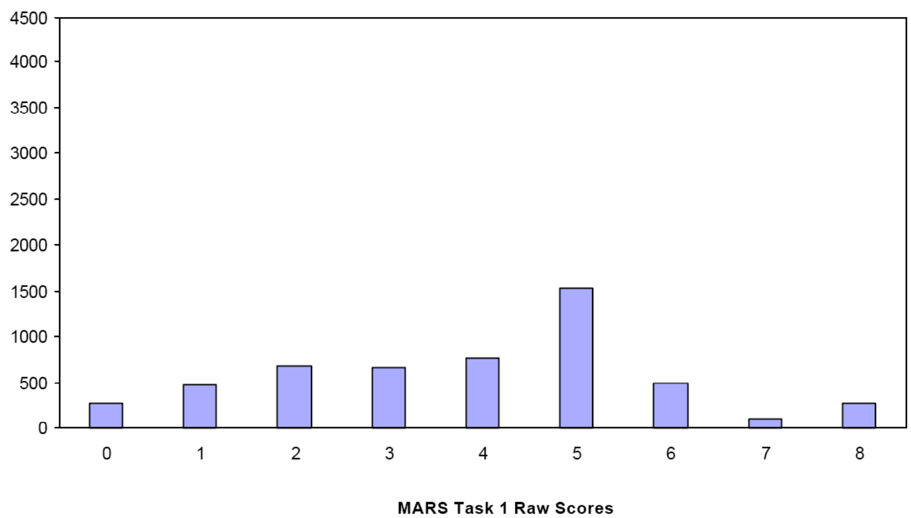
Areas of difficulty for seventh graders:
- Finding sample space for a combined event, like finding the total of 2 die.
- Finding the number of possible outcomes for a combined event.
- Distinguishing between a number on a die and a desired combination or outcome.

Strategies used by successful students:
- More complete definitions for part 2b, using words like total possibilities instead of the numbers on the die.
- Making an organized list or diagram to define the sample space in part 3

Table 35: Frequency Distribution of MARS Test Task 1, Grade 7

Task 1 Scores	Student Count	% at or below	% at or above
0	278	5.3%	100.0%
1	481	14.5%	94.7%
2	672	27.3%	85.5%
3	661	39.9%	72.7%
4	771	54.6%	60.1%
5	1520	83.6%	45.4%
6	495	93.0%	16.4%
7	97	94.9%	7.0%
8	269	100.0%	5.1%

Figure 44: Bar Graph of MARS Test Task 1 Raw Scores, Grade 7



The maximum score available for this task is 8.
The minimum score needed for a level 3 response, meeting standards, is 4 points.

Most students, 86%, could use words to describe the probability of simple events. More than half the students, 60%, could use words to describe two of the three events in part one and could write numerical probabilities for events with one standard die and explain how they figured it out in part two. 5% of the students could meet all the demands of the task including finding the sample space for rolling two dice and finding the number of favorable outcomes. 5% of the students scored no points on this task. All the students in the sample with this score attempted the task.

Will it Happen?

Points	Understandings	Misunderstandings
0	All the students in the sample with this score attempted the task.	14% of the students thought it was unlikely rather than impossible to run into President Lincoln. 10% of the students thought that it was likely or unlikely to get heads when tossing a coin. Students had trouble attaching quantitative thinking to their word choices.
1	Students could describe probabilities in words for 2 of the 3 choices in part 1.	Students continued to use word descriptions in part 2. 11% of the students thought rolling a 4 was unlikely. 5% thought it was equally likely to roll a 4 and another 5% thought it was likely to roll a 4.
2	Students could use words to describe probabilities in part 1.	About 5% of the students thought that there was a 4/6 or 2/3 chance of rolling a 4. 8% of the students thought that it was equally likely to get an odd number. They did not attempt to write a numerical probability. 8% thought it was likely to roll an odd number.
4	Students could use words to describe 2 of the probabilities in part 1. They could also describe events with a single die in numerical terms and describe how they figured it out.	Students struggled with combined probabilities in part 3. 11% still used words to describe the probability in part 3.
6	Students could use words and numbers to describe probabilities in part 1 and 2. Students realized that the numbers 9 + 11 equaled the desired outcome of 20.	Students did not know how to define the sample space for the total number of possible outcomes. 10% thought the probability was 1/12. 11% thought the probability was 2/12. They were thinking about 12 numbers on the two die, rather than the results of one die. Many students wrote a probability with 6 in the numerator, because there are 6 numbers on a die. Some students had difficulty distinguishing between the numbers on the die and the outcomes of adding; so two numbers are needed to make 20: 4/12, 2/6 or 1/3.
8	Students could use words and numbers to describe simple probabilities. Students could find the sample space for a combined event and find the number of favorable outcomes to write a probability.	

Implications for Instruction

Students in fourth grade learn to use words to describe the "likelihood" of an event. Some students at this grade level are using these words with a more common usage than with a quantitative sense. For example, students might say that it is likely to roll a 4 on a die, because it is something that could happen rather than thinking there are more numbers on a die that aren't 4 than are 4. Many students do not know how to use numbers to write a probability. Probability should be taught for mastery in sixth grade and students are held accountable for this knowledge on the high school exit exam. Because of the changing games used by students today, some students do not even understand the context of rolling dice. For example Student H thought both die would generate the same numbers. Teachers at later grades need to work these concepts into the curriculum to make sure ideas are fully developed. For example, many students had trouble distinguishing the numbers on the dice from the sample space when thinking about combined probabilities. Students also need to know the difference between the actual numbers on the dice and the numbers, which can be combined to get a favorable outcome. Example: If a nine and an eleven are needed to make an outcome of 20, that is 2 numbers on the die, but one event or outcome. Students need to go through several phases of understanding probability, from the very concrete level of understanding how rolling dice work and how to combine the results, to a pictorial level being able to make an organized list or diagram to show the sample space, to an abstract level of using formulas to calculate probabilities. Students, who had more complete definitions of what the two numbers are that make up a numerical probability, seemed to do a better job of calculating a combined probability. The students need to be engaged in rich discussions to bring out some of these subtleties in writing probabilities that are not necessarily apparent when acquiring the procedural knowledge. Some students tried to apply the knowledge or procedures for calculating the probability for an event(s) on one die to finding the probability of combined event.

Ideas for Action Research: Re-engagement

One useful strategy when student work does meet your expectations is to use sample work to promote deeper thinking about the mathematical issues in the task. In planning for re-engagement it is important to think about what is the story of the task, what are the common errors and what are the mathematical ideas that students need to think about more deeply. Then look through student work to pick key pieces of student work to use to pose questions for class discussion. Often students will need to have time to rework part of the task or engage in a pair/share discussion before they are ready to discuss the issue with the whole class. This reworking of the mathematics with a new eye or new perspective is the key to this strategy.

To plan a follow-up lesson using this task, pick some interesting pieces of student work that will help students confront and grapple with some of the major misconceptions. Make the misconceptions explicit and up for public debate.
During the discussion, it is important for students to notice and point out the errors in thinking.

I might start the lesson, by trying to develop an agreed definition for writing a numerical probability to help bring all students to common ground for thinking about the complexities later on. I might start by posing the question:

In class we have been trying to improve how we explain our thinking. Here is one response I saw.

" 3/6 Well I counted the number of odd numbers between 1,2,3,4,5, and 6."

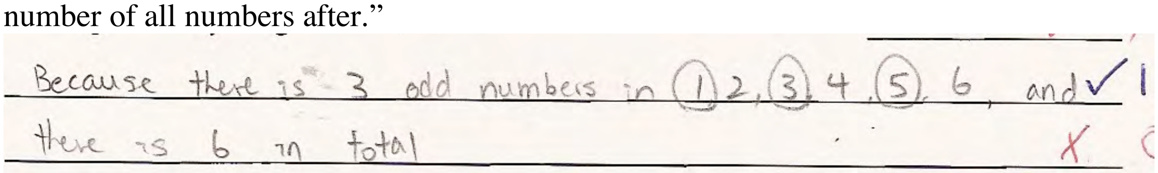
Is this information clear? What information might make this clearer?

I would hope that students would discuss where the 3 came from, where the 6 came from. Then I might give students some solutions to compare to help them build an internal rubric for good explanations. I might pose the question:

Here are some other explanations. What is each person thinking? Where do their numbers come from? Are there some parts that could be improved? If so, how?

"I know that there are six numbers and only three of them are odd. So, I put the three over six and reduced, to get 1/2."

"3/6 First I counted out how many odd numbers there was and then I counted the number of all numbers after."


Because there is 3 odd numbers in 1, 2, 3, 4, 5, 6, and there is 6 in total

"There are 3 even numbers & 3 odd numbers. 3 out of 6 = 1/2."

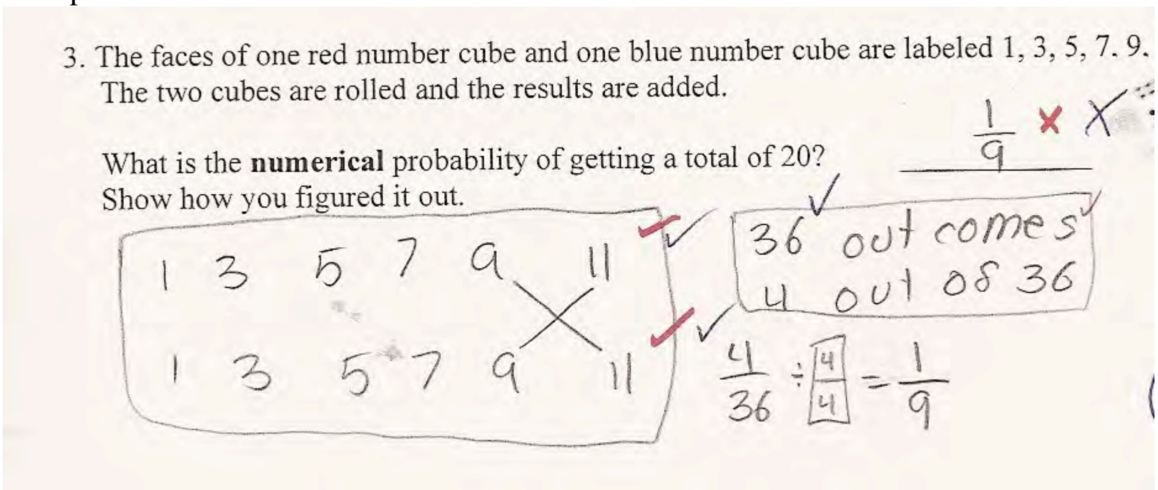
After spending time on digging into this basic understanding of probability, I think the class would be ready to dig into the more complex issues in part 3. I might start with the issue of numbers on the die versus outcomes. I might ask:

Jeff wrote 1, 3, 5, 7, 9, 11 9 + 11 = 20, so the probability if 2:6. *What do you think Jeff was thinking? What might he be confused about?*

I am hoping the students will also bring up that Jeff is only looking at one die not two. Then I would look at a sample space 12.

Sarah put 1/12. Only 9 + 11 = 20 There are 2 dice and 6 numbers on each dice.

Fred put:



3. The faces of one red number cube and one blue number cube are labeled 1, 3, 5, 7. 9. The two cubes are rolled and the results are added.

What is the **numerical** probability of getting a total of 20? $\frac{1}{9}$

Show how you figured it out.

1 3 5 7 9 11

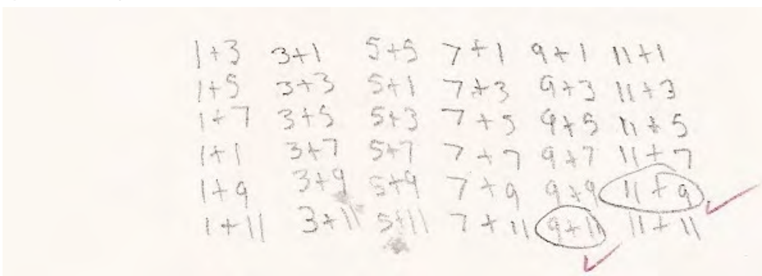
1 3 5 7 9 11

36 out comes

4 out of 36

$\frac{4}{36} \div \boxed{\frac{4}{4}} = \frac{1}{9}$

Gena wrote:



1+3	3+1	5+5	7+1	9+1	11+1
1+5	3+3	5+1	7+3	9+3	11+3
1+7	3+5	5+3	7+5	9+5	11+5
1+1	3+7	5+7	7+7	9+7	11+7
1+9	3+9	5+9	7+9	9+9	11+9
1+11	3+11	5+11	7+11	9+11	11+11

What is each person thinking? Where do their numbers come from? Can you use this thinking to make a correct solution?